

NAG C Library Function Document

nag_quartic_roots (c02alc)

1 Purpose

nag_quartic_roots (c02alc) determines the roots of a quartic equation with real coefficients.

2 Specification

```
void nag_quartic_roots (double e, double a, double b, double c, double d,
                        double zeror[], double zeroi[], double errest[], NagError *fail)
```

3 Description

nag_quartic_roots attempts to find the roots of the quartic equation

$$ez^4 + az^3 + bz^2 + cz + d = 0,$$

where e , a , b , c and d are real coefficients with $e \neq 0$. The roots are located by finding the eigenvalues of the associated 4 by 4 (upper Hessenberg) companion matrix H given by

$$H = \begin{pmatrix} 0 & 0 & 0 & -d/e \\ 1 & 0 & 0 & -c/e \\ 0 & 1 & 0 & -b/e \\ 0 & 0 & 1 & -a/e \end{pmatrix}.$$

Further details can be found in Section 6.

To obtain the roots of a cubic equation, nag_cubic_roots (c02akc) can be used.

4 Parameters

- | | | |
|----|---|---------------|
| 1: | e – double | <i>Input</i> |
| | <i>On entry:</i> e , the coefficient of z^4 . | |
| | <i>Constraint:</i> $e \neq 0.0$. | |
| 2: | a – double | <i>Input</i> |
| | <i>On entry:</i> a , the coefficient of z^3 . | |
| 3: | b – double | <i>Input</i> |
| | <i>On entry:</i> b , the coefficient of z^2 . | |
| 4: | c – double | <i>Input</i> |
| | <i>On entry:</i> c , the coefficient of z . | |
| 5: | d – double | <i>Input</i> |
| | <i>On entry:</i> d , the constant coefficient. | |
| 6: | zeror[4] – double | <i>Output</i> |
| 7: | zeroi[4] – double | <i>Output</i> |
| | <i>On exit:</i> zeror $[i - 1]$ and zeroi $[i - 1]$ contain the real and imaginary parts, respectively, of the i th root. | |

- 8: **errest[4]** – double *Output*
On exit: **errest**[$i - 1$] contains an approximate error estimate for the i th root.
- 9: **fail** – NagError * *Input/Output*
 The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

NE_REAL

On entry, **e** = 0.0.
 Constraint: **e** $\neq$ 0.0.

NE_C02_OVERFLOW

The companion matrix H cannot be formed without overflow.

NE_C02_NOT_CONV

The iterative procedure used to determine the eigenvalues has failed to converge.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments

The method used by the routine consists of the following steps, which are performed by routines from LAPACK.

- (a) Form matrix H .
- (b) Apply a diagonal similarity transformation to H (to give H').
- (c) Calculate the eigenvalues and Schur factorization of H' .
- (d) Calculate the left and right eigenvectors of H' .
- (e) Estimate reciprocal condition numbers for all the eigenvalues of H' .
- (f) Calculate approximate error estimates for all the eigenvalues of H' (using the 1-norm).

6.1 Accuracy

If **fail.code** = **NE_NOERROR** on exit, then the i th computed root should have approximately $|\log_{10}(\mathbf{errest}[i - 1])|$ correct significant digits.

6.2 References

Golub G H and Van Loan C F (1996) *Matrix Computations* Johns Hopkins University Press, Baltimore (3rd Edition)

7 See Also

nag_cubic_roots (c02akc)

8 Example

To find the roots of the quartic equation

$$z^4 + 2z^3 + 6z^2 - 8z - 40 = 0.$$

8.1 Program Text

```

/* nag_quartic_roots (c02alc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <nagc02.h>

int main(void)
{
    double a, b, c, d, e;
    double *errest=0, *zeroi=0, *zeror=0;
    Integer i;
    Integer exit_status=0;
    NagError fail;

    INIT_FAIL(fail);
    Vprintf("c02alc Example Program Results\n\n");
    if
    (
        ! (errest = NAG_ALLOC(4,double)) ||
        ! (zeroi = NAG_ALLOC(4,double)) ||
        ! (zeror = NAG_ALLOC(4,double))
    )
    {
        Vprintf ("Allocation failure\n");
        exit_status=-1;
        goto END;
    }

    /* Skip heading in data file */
    Vscanf("%*[\n]");
    Vscanf("%lf %lf %lf %lf %lf", &e, &a, &b, &c, &d);

    c02alc (e, a, b, c, d, zeror, zeroi, errest, &fail);
    if (fail.code == NE_NOERROR)
    {
        Vprintf(" Roots of quartic equation           Error estimates\n");
        Vprintf("                               (machine-dependent)\n\n");
        for (i = 0; i <= 3; ++i)
        {
            Vprintf("%s %10.5f %10.5f%s           %g\n", " z =",
                    zeror[i], zeroi[i], "*i", errest[i]);
        }
    }
}

```

```

else
{
    Vprintf("Error from c02alc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
END:
if (errest) NAG_FREE (errest);
if (zeroi)  NAG_FREE (zeroi);
if (zeror)  NAG_FREE (zeror);
return exit_status;
}

```

8.2 Program Data

c02alc Example Program Data

1.0 2.0 6.0 -8.0 -40.0 : Values of e, a, b, c and d

8.3 Program Results

c02alc Example Program Results

Roots of quartic equation			Error estimates (machine-dependent)
z =	2.00000	0.00000*i	3.38974e-15
z =	-2.00000	0.00000*i	5.29396e-15
z =	-1.00000	3.00000*i	4.54379e-15
z =	-1.00000	-3.00000*i	4.54379e-15
